expansion of $\Lambda_n(z)$ [6], i.e.,

$$\Lambda_n(z) \cong 1 - \frac{z^2}{4(n+1)} + \frac{z^4}{32}\frac{n!}{(n+2)!} - \frac{z^6}{384}\frac{n!}{(n+3)!} \tag{A-7}$$

(A-6) can be written as

$$I_2 \cong \frac{ie^{i(k_0-k)(L-x'+ct')}}{(k_0-k)} \exp\left[-i\frac{(\kappa_y^2+\kappa_z^2)}{2(k_0-k)} \cdot (L-x'+ct')\right] \alpha_\Lambda(\kappa,x',t'). \tag{A-8}$$

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Coherent Electromagnetic Losses by Scattering from Volume Inhomogeneities

LADISLAV E. ROTH AND CHARLES ELACHI

Abstract—**A parametric analysis of the coherent electromagnetic losses due to scattering from volume weak inhomogeneities is reported. It is shown that even in the case of very weak inhomogeneities, the loss could be appreciable in a resonance region and thus play a more dominant role than conductivity losses.**

I. Introduction

Wave propagation in a layered conducting medium has been studied in detail by many authors [1], [2] and applied to geologic subsurface sounding. However, in most previous studies, the losses due to finite scatterers were not included. In this short paper, we use the mathematical analysis and results of Karal and Keller [3], [4] to determine the losses encountered by a coherent wave due to weak random dielectric inhomogeneities in a conducting medium as a function of the wavelength, the inhomogeneities size, and their magnitude. We found that in a wide frequency band, the scattering losses could be many times larger (more than an order of magnitude in some cases) than the conductivity losses. Thus due consideration must be given to subsurface inhomogeneities (if they exist) in the design of coherent sounding experiments, using synthetic aperture techniques [5] especially at high operating frequencies (i.e., 10 MHz and higher).

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The authors are with the Jet Propulsion Laboratory, California Institute of Technology, Pasadena, Calif. 91103.

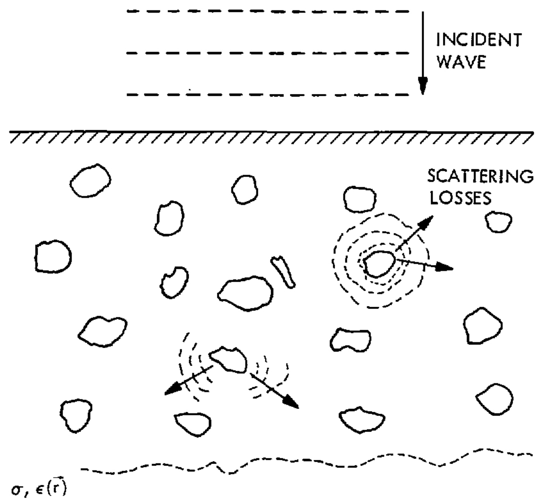

Fig. 1. Geometry: sounding plane wave loses energy due to conduction and scattering.

The loss encountered by the coherent component of the wave would lead to blurring of the image which is formed by the synthetic aperture. The results are also applicable to optical scattering losses from localized inhomogeneities in optical waveguides, holograms, and atmospheric channels.

We will consider only the case of electromagnetic waves. However, the theory of Karal and Keller could be similarly used for the case of elastic waves.

II. Approach, Results, and Interpretation

Let us consider a medium (Fig. 1) of conductivity σ, permeability μ, and dielectric constant $\varepsilon(r)$ given by

$$\varepsilon(r) = \bar{\varepsilon}[1 + \eta f(r)] \tag{1}$$

where $\bar{\varepsilon}$ is the average dielectric constant, η is a small parameter ($\eta \ll 1$), $f(r)$ is a normalized random function, and r is the three-dimensional position variable. The dispersion equation for the propagation constant k of a transverse plane wave travelling in such a random weakly inhomogeneous medium was derived by Karal and Keller and is given by

$$k^2 - k_0^2 - \eta^2 D(k) = 0 \tag{2}$$

where $k_0^2 = \mu\bar{\varepsilon}\omega^2 + i\mu\sigma\omega$; k_0 is the wavevector for $\eta = 0$, and $D(k)$ is given by [4],

$$D(k) = (\mu\bar{\varepsilon}\omega^2)^2 \int_0^\infty R_{\varepsilon\varepsilon}\left[G_1 f - G_2 \frac{\partial f}{\partial k}(r^2k)^{-1}\right] dr \tag{3}$$

$$f = 4\pi r \sin kr/k$$

$$G_1 = (-1 + ik_0 r + k_0^2 r^2)e^{ik_0 r}(4\pi k_0^2 r^3)^{-1} - \delta(r)/12\pi k_0^2 r^2$$

$$G_2 = (3 - 3ik_0 r - k_0^2 r^2)e^{ik_0 r}/4\pi k_0^2 r^3$$

$$R_{\varepsilon\varepsilon} = \langle f(r) f(r')\rangle = \text{autocorrelation function.}$$

The two functions G_1 and G_2 are the components of the Green tensor. The dispersion relation (2) has to be solved numerically for a specific autocorrelation function. The integral was first evaluated analytically. To illustrate, we considered the exponential case

$$R_{\varepsilon\varepsilon} = e^{-r/r_0}, \tag{4}$$

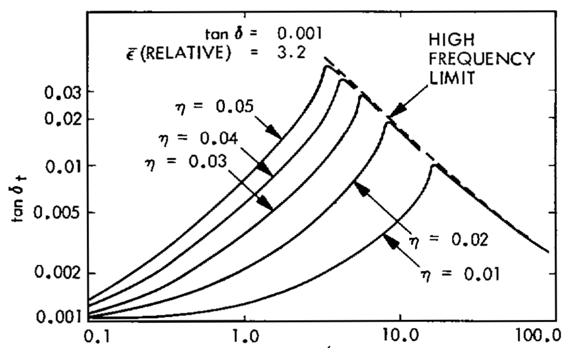

Fig. 2. Total loss tangent as function of relative size of scatterers. $\bar{\varepsilon}$ (relative) = 3.2, tan δ = 0.001. Dashed curve is high-frequency limit.

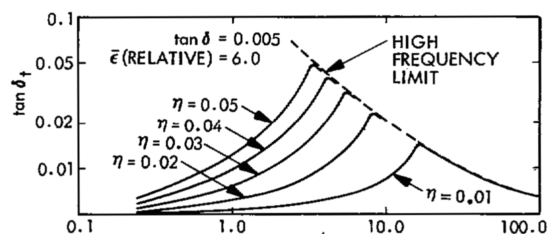

Fig. 3. Total loss tangent as function of relative size of scatterers. $\bar{\varepsilon}$ (relative) = 6.0, tan δ = 0.005. Dashed curve is high-frequency limit.

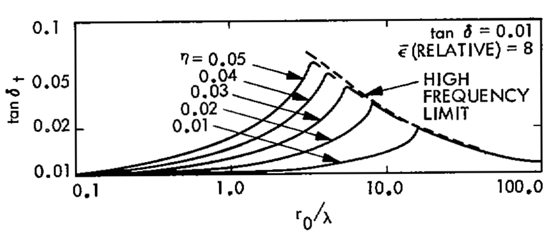

Fig. 4. Total loss tangent as function of relative size of scatterers. $\bar{\varepsilon}$ (relative) = 8.0, tan δ = 0.01. Dashed curve is high-frequency limit.

where r_0 represents the correlation distance, and we solved the dispersion relation to determine the effective total loss tangent tan δ_t as a function of r_0/λ for different values of $\bar{\varepsilon}$ and the conductivity loss tangent tan δ. The effective total loss tangent tan δ_t is equal to the sum of the conductivity loss tangent tan δ and the scattering loss tangent. The wavelength λ in the dielectric is equal to $\lambda_0\sqrt{\varepsilon_0\bar{\varepsilon}}$, where λ_0 is the wavelength in vacuum. In Figs. 2, 3, and 4 we have plotted three typical sets of curves for different values of η. The value $\bar{\varepsilon} = 3.2$ is representative of the lunar soil [6] in the frequency range of the Apollo 17 sounder, $\bar{\varepsilon} = 6$ corresponds to compact siliceous metamorphic rocks [7], [8], and $\bar{\varepsilon} = 8$ is a realistic value for the compact unbreciated mafic and ultramafic igneous rocks.

From a practical point of view, r_0 represents an average of the radius of the volume scatterers. The quantity η can be loosely interpreted as the product of the fractional change in dielectric constant and the fractional volume representing the inhomogeneity,

$$\eta = \frac{\Delta\varepsilon}{\bar{\varepsilon}}\frac{\Delta V}{V} = \frac{\Delta\varepsilon}{\bar{\varepsilon}}\frac{4}{3}\pi r_0^2 N,$$

where N is the number of scatterers per unit volume.

The main conclusion which results from the curves in Figs. 2, 3, and 4 is that the total loss (scattering plus conductivity) can be large relative to the conductivity loss alone. The scattering loss is large in a general region of $0.1 < r_0/\lambda < 100$, the exact boundaries being functions of all parameters. In the high- and low-frequency limits, tan $\delta_t \to$ tan δ. As we would expect, larger values of η will in general lead to larger losses. However for high frequencies, the loss angle tends toward a limiting curve which is independent of η. The value of η only enters in the determination of the lower limit of this so-called "high frequency region." This fact was specifically pointed out by Karal and Keller [3], when they showed that for $r_0/\lambda > 1/2\pi\eta$, the wave vector is given by

$$k = k_0\left(1 + \frac{\eta}{2} + i\frac{\lambda}{4\pi r_0}\right)$$

which corresponds to a loss angle of

$$\tan\delta_t = \tan\delta + \frac{\lambda}{2\pi r_0}.$$

This high-frequency limit is shown in the figures. We also found that, to the left of the peaks, (tan δ_t − tan δ) is proportional to η^2.

A second factor of interest is that the peaks of the loss curves, for a given η, occur at the same value of r_0/λ regardless of the value of tan δ and $\bar{\varepsilon}$, except that $\bar{\varepsilon}$ enters into the determination of λ. In other terms, for a given value of η, the peak is determined by the size of the scatterers relative to the wavelength in the medium. We also note that the position of the peak is proportional to $1/\eta$.

To illustrate, let us consider the case where $\bar{\varepsilon}$ (relative) = 3.2, tan δ = 0.001, λ_0 (in vacuum) = 2 m, r_0 = 1.4 m, $\Delta\varepsilon/\bar{\varepsilon} = 0.1$, and $N = 0.05\ \mathrm{m}^{-3}$. This gives $\Delta V/V = 0.21$ (i.e., there is one scatterer of volume 4.2 m^3 in a volume of 20 m^3), and $\eta = 0.04$. The corresponding total loss tangent is

$$\tan\delta_t = 0.006 = \begin{cases} 0.001, & \text{conductivity loss} \\ 0.005, & \text{scattering loss.} \end{cases}$$

Thus the scattering loss is the dominant factor.

III. Conclusion

We have numerically determined and plotted the losses due to random weak inhomogeneities and have shown that under certain sets of conditions, the scattering loss could be larger than the conductivity losses.

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